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ABSTRACT

This paper argues that the introduction of the scientific method in the very rich environments of the natural sciences or human sciences may disguise the process and create difficulties for students because of the multiplicity of variables involved, whereas the variables present in a mathematical context can be readily manipulated and their numbers are easily controlled. Other advantages to modeling the scientific method in mathematics are identified as the analogy between the processes used in certain mathematical problems and the more generic processes of the scientific method, as many of the steps employed in obtaining solutions to problems pattern the behavior used in scientific investigation, e.g., simplification, keeping records and collecting data, looking for patterns, hypothesizing and testing, and finally generalizing. (It is pointed out that all of these activities are part of a more general process known as induction.) The characteristics of a model are juxtaposed with mathematics problems to demonstrate the value of using mathematics problems as a model for scientific inquiry, and several sample problems are worked through to demonstrate the process. It is concluded that such problems are particularly effective in leading students to practice induction in a manner similar to that used in scientific inquiry. (DJR)

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Highlighting The Scientific Method Through Mathematics Problems

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While the use of the "scientific method" is influenced by the setting in which it is being employed, some key notions are always involved when one is using the scientific method of inquiry. These notions include observing, questioning, inferring, keeping records of data, hypothesizing and generalizing. All these activities are a part of a more general process known as induction. Induction may include many observations and experiments. In the context of natural sciences such as biology or chemistry or human sciences such as psychology or sociology, the scientific method is pursued in a very rich environment. This means that a multiplicity of variables impinge on the phenomena being studied. For those being initiated into the scientific enterprise, that is students, this may create difficulty and disguise the process because of the effort to control all but pertinent variability. The thesis of this paper is that virtue exists in modeling the scientific method in a less variable-rich environment, namely mathematics.

Before looking at some examples of activities that constitute such modeling, we will look in more detail at

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what a model is. A model is a representation or partial account of something. The model must be separated from what is being modeled. There is similarity in structure between the model and what is being modeled, but there are also differences, usually obvious, between the two. Some characteristics that models may possess include

i. The model often simplifies the situation

ii. A model is often ideal

iii. A model may be discarded when its usefulness is ended (Richards, 1979).

The concept of model is used here in the broad sense of representing the process of scientific enquiry. The processes used in certain mathematical problems are analogous to the more generic processes of the scientific method. Many of the steps employed in obtaining solutions pattern the behavior used in scientific investigation. These include simplification, keeping records and collecting data, looking for patterns, hypothesizing and testing, and finally generalizing. Negatively, one must realize that the mathematical setting used lacks the complexity of settings for natural or human sciences. The variables present in a mathematical context can be readily manipulated and their numbers easily controlled while in other contexts this manipulation and control may be quite difficult or impossible.

As a closing support for the use of mathematics problems as a model for scientific inquiry, let us

juxtapose the characteristics of a model listed earlier with mathematics problems. First, mathematics simplifies the situation because it shields the inquiry from the intrusion of extraneous variables or experimental "noise". Hence it is a "cleaner" environment for investigation, and one has reason to expect that the process will be more clearly etched. Mathematics acts as an ideal model in that it presents characteristics that should be copied or repeated whenever possible. The idea that a model is transient applies to mathematics problems in that the problems dealt with may be simple and of no long range interest except for the process involved in the solution.

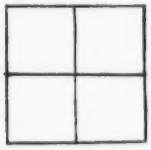
Example Problems

In order to make more concrete what has been discussed in the preceding paragraphs, two problems will be shown with emphasis on the analogy of the processes used for solution and the scientific method.

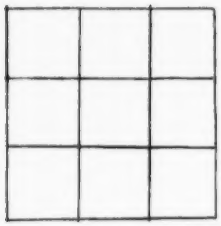
Problem 1: How many squares are on a checkerboard? (i. e. an 8 x 8 square grid)

Some will answer immediately that there are 64. There are at least that many counting only the 1 x 1 squares. But observe there are 2 x 2's, 3 x 3's, 4 x 4's, etc. And those are partially overlapping in many ways. Hence, we conclude that this problem is not too easy. In fact, the 8 x 8 dimension may be causing much of the difficulty. If the grid was a 2 x 2 or 3 x 3, it would be much easier

to count the squares.



5 squares



14 squares

The solution evolves by working with the simpler related grids, keeping records of the data, looking for a pattern and ultimately observing that the number squares on the original grid is

$$8^2+7^2+6^2+5^2+4^2+3^2+2^2+1^2$$

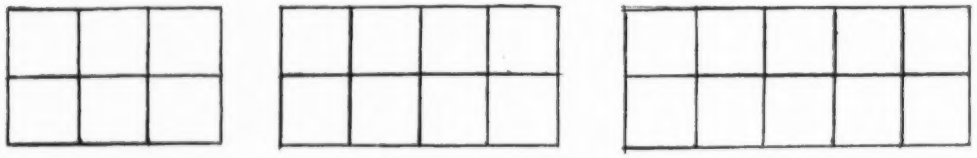
This solution has been found by an instinctive process that is analogous to processes of natural science and human science. The solution can be readily generalized to an N x N grid by noting that the number of squares is

$$N^2+(N-1)^2+\ldots+3^2+2^2+1^2$$

Problem 2: How many squares are on a rectangular grid?

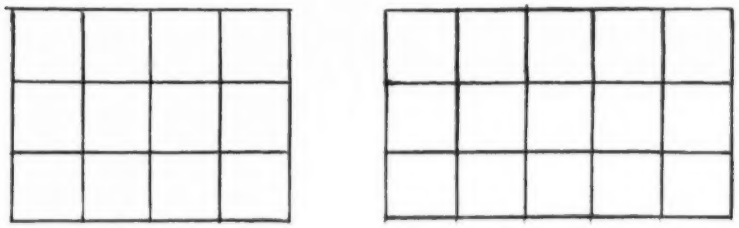
While problem 1 had only one variable, the dimension of the square grid, this problem has two variables, namely the length and width of the rectangular grid. Hence to attack this problem in a manner similar to that used on problem 1 will require controlling variables. That is, hold one dimension constant and let other vary in a systematic way. It may be necessary to repeat the process several times to pick up a pattern. For example, hold the

width constant at two and let the length range over three, four, and five, and count squares.



6 + 2 8 + 3 10 + 4

Now increase the width by one to three and let the length be four and five and count squares



12 + 6 + 2 15 + 8 + 3

Continuing observation and analysis of such examples leads to the observation of the pattern that the number of squares on a 4 x 6 grid is

24 + 15 + 8 + 3 = 4x6 + 3x5 + 2x4 + 1x3 = 50

and in general the squares on an M x N grid is given by the following calculation.

MxN+(M-1)(N-1)+(M-2)(N-2)+(M-3)(N-3)+... until it is not possible to continue. This problem again highlights the strategies of simplifying, data colection, looking for patterns, hypothesizing and generalizing. In addition, it also includes the idea of controlling variables and letting others vary in a systematic fashion.

Additional Problems

1. I am building steps with blocks in the following way

How many blocks will it take to build 10 steps? How about 20 steps? How about N steps?

2. What is the maximum number of chords that can be drawn on a circle by placing 10 points on the circle? What if you put 25 points on the circle? How about N points?

3. What is the minimum number of weighings (barring luck) using an equal arm balance to isolate one counterfeit coin in ten? The counterfeit is lighter than the others. What about one counterfeit in 30 coins? How about 198 coins? How about N coins?

Conclusion

In summary, the problems outlined and those suggested in the preceding paragraphs have merit in themselves for use in developing problem solving skills. In addition, they and the many others like them are particularly effective in leading students to practice induction in a manner similar to that employed in scientific inquiry.

Reference

Richards, J. "Modelling numbers and reality". A paper presented at the Symposium on Models of Mathematical Cognitive Development, University of Georgia, Athens, Georgia, 1979.